Joint IMD-WMO group fellowship Training

On

Numerical Weather Prediction

Through

Distance learning mode

Managed by

Meteorological Training Institute, India Meteorological Department (IMD), Pune

During

04 Oct to 10 Nov 2021

Lecture Notes

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1. Basic concept of an Initial value problem:

A system which has a time evolution, i.e., which varies with time is broadly known as a dynamical system. It is desired to have a prediction of a dynamical system. Prediction of a dynamical system requires a rule or rules governing the time evolution of the dynamical system and complete information of the same at its current state. As atmospheric condition also varies with time, so it is also a dynamical system and accordingly its prediction requires complete information about the current/present state of the atmosphere and a rule for its time evolution.

In 1904, V. Bjerknes first realized that problem of predicting the atmospheric condition (weather forecast) is an initial value problem (IVP). IVP is mathematically expressed as: $Solve\ \frac{\partial \varphi}{\partial t} = f(x;t)\ ; given\ \varphi(x,t_0) = \varphi_0(x), for\ \varphi(x;t) \quad for\ all\ x\ \&\ t > t_0\ where\ \ f(x;t) is\ a\ known\ quantitity$. Equation is the mathematical form of the rules for time evolution of the atmosphere dynamical system and the condition $\varphi(x,t_0) = \varphi_0(x)$ represents the mathematical form of current state of the same called initial condition (IC). If a very small change in the IC results in a corresponding small change in the forecast, then we say the IVP is properly or well posed, otherwise improperly or ill posed. Many times it has been observed that a slight change in the initial condition results in large change in the forecast, which may be attributed to the non-linearity and improper representation of physical processes.

In mathematics, in the field of differential equations, a boundary value problem is a differential equation together with a set of additional constraints, called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions.

Example: Solve $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = g(x,y)$, where, $g(x,y)$ is a known function and given the following boundary condition:

$$\varphi(x,-b) = \varphi_{-b}, \varphi(x,b) = \varphi_b, \varphi(-a,y) = \varphi_{-a}, \varphi(a,y) = \varphi_a$$

As the problem of forecasting weather requires the complete information about the present state of the atmosphere, hence it is essentially an IVP. At the same time as the weather condition at a place at a given time is also dependent on the weather condition over the surrounding of the place also. So, this is a BVP also. Thus, problem of forecasting weather at a place is essentially an IVP as well as a BVP.

2. Historical Background NWP and hierarchy of NWP models:

From the historical point of view, V. Bjerknes was the first scientist who suggested in 1904 that the atmospheric prediction problem is an initial value marching problem. The first practical attempt to solve the set of equations that govern the atmospheric motions numerically was made by Lewis F. Richardson during the first world war. He used a desk calculator to compute the surface pressure tendencies at two grid points. Unfortunately, his results were in error by an order of magnitude and were totally unacceptable. As a result, his monumental work was ignored for more than two decades. The failure of Richardson's numerical treatment was, at that time, attributed to poor initial data available, especially the absence of upper air data. Later, it was discovered that the atmospheric equations in its complete form, so called 'primitive' form, admit solutions corresponding to not only the slow-moving atmospheric waves (Rossby Waves) but also fast-moving sound and gravity waves. These high-speed waves amplify spuriously with the time and mask the solutions relating to atmospheric waves if not properly controlled. It was theoretically shown by Courant, Friedrichs and Lewy in 1928 that the space and time increments used in the marching scheme have to satisfy the stability criterion given by

$C (\Delta t/\Delta x) \leq 1$

where C = speed of fastest wave

Δt = time increment

Δx = space increment

Later, in 1948, Charney showed that by making use of hydrostatic and geostrophic assumptions the high-speed sound and gravity waves can be effectively 'filtered'. In 1950, using the first electronic calculator ENIAC and the filtered model, Charney, Fjortoft and Von Neumann produced the first successful numerical prediction.

Since then, there has been a rapid progress in all phases of NWP. These improvements are mainly due to considerable increase in the quantity of meteorological data advances in telecommunication system tremendous progress in computer technology and development of much better and sophisticated numerical models.

Even then the success of NWP is relatively poor in tropics compared to that in extra tropical region due to certain constraints peculiar to tropics. The major problems in the tropical belt are:

Sparse data network, as the region is dominated by the oceans, deserts and mountains from where observations on regular basis are poor and hence, difficulty in defining an accurate initial state.

Weak wind-pressure balance (small coriolis force and weak pressure gradient).

Dominance of meso-scale systems like cumulus convection, thunderstorms etc. which are known to supply energy to large scale systems like cyclones, depressions etc. but difficult either to detect by synoptic network or forecast well in advance.

Lack of a precise understanding and modelling of various physical processes like air-sea interaction, boundary layer forcing, radiation, convection etc.

Fortunately, in the modern days with the advances in observational technology, telecommunication system, computing powers and understanding of different physical processes there is much needed progress in the field of NWP and the reliability of numerical forecasts is much higher compared to that of a few decades earlier.

Barotropic models do not allow temperature advection because wind is parallel to isotherms and hence, can't forecast the development of new weather systems. They, in fact only extrapolate the system by advection of vorticity. For more accuracy and reliability, prediction of development of new systems is essential. For this purpose, thermal advection has to be included in the numerical model and hence more than one level is required to be incorporated in the model, i. e. one has to consider a baroclinic model. Accordingly 2-layer Baroclinic model (by Charney and Eady) developed.

The two level baroclinic model has not proved to be a very successful prediction model primarily because it tends to produce stronger baroclinic development than observed in many cases. This was attributed to the fact that the heights of only two widely separated isobaric surfaces reveal very little of the detailed vertical structure of the atmospheric flow. However, the simplicity of this model does make it a useful tool for analysis of the physical processes occurring in baroclinic disturbances-extratropical frontal systems. The two level baroclinic model can be extended to multi-level model for greater vertical resolution and improved weather forecast.

Barotropic and 2 or multi-layer baroclinic models are known as filtered model. However, filtered models, though filter out gravity waves and become easier to manage by numerical schemes, have a definite limit on their accuracy due to the various approximations made. Further, these models can't be used in low latitudes where quasi-geostrophic assumption is not valid. Under these circumstances, it was thought that direct utilisation of basic hydrodynamic equations in their primitive form might improve the accuracy of numerical models in general. The availability of much faster computers and improved understanding of computational problems made it possible to carryout time integration for longer period of the basic primitive equations using small time space. The first successful experiments using primitive equations were carried out by Hinkelman in 1959. Since then NWP has marched steadily forward and now a day's primitive equations models are widely utilised world over for operational numerical weather prediction in different time scales.

3. Limitation in traditional synoptic method, demand for a location and time specific forecast system, giving rise to the concept NWP.

The main goal of a Meteorological office is to issue weather forecasts in different time scales to various user agencies like aviation, marine, agriculture, water management, builders, tourism industry, planners etc. and to public. The forecast requirement varies from detailed weather forecasts in time scales of a few hours to days and to more general indication of the broad weather patterns of succeeding months, seasons or even beyond. For example, aviation industry needs weather information in time scales of a few hours or a day, whereas agriculture sector demands weather forecasts in time scales of a week. Weather forecasts of a month or a season in advance are required mostly by planners. Thus, the weather forecasts are broadly classified as (i) short-range (forecast validity upto 3 days), (ii) medium range (forecast validity beyond 3 days upto 10 days) and (iii) long-range (forecast validity beyond 10 days or a few weeks or a month or a season or even beyond).

Weather forecasting basically consists of two steps. The first step is to have an accurate assessment of the present/initial state of the atmosphere. This helps in identifying the different weather systems and their horizontal and vertical state. As we know, there are a variety of phenomena occurring in the atmosphere having different space and time scales. The characteristic sizes of these motions vary from a fraction to centimetre to several thousands of kilometres, with time scales of a fraction of a section to several weeks. Each of the various scales of motions has a varying degree of influence upon all the others and it is important to properly observe, analyse and account them in atmospheric studies and weather forecasting. As weather has no political boundary, one needs weather data from a fairly large region; the area from which data required increases with the duration of forecast made, since weather systems from one part of a region may travel and affect the weather condition over a far-off region in course of time.

The second step in weather forecasting is to utilise a suitable technique to predict the future state of the atmosphere. Primarily, there are three methods (i) synoptic method (ii) statistical method and (iii) numerical weather prediction (NWP). Traditional Synoptic method is a suite of following tasks:

- Taking observation over the entire globe simultaneously at some fixed time (expressed in terms of GMT/UTC), known as Synoptic hours of observation.
- Preparation of coded message for the observed information.
- Exchange of coded messages among all centers for weather forecasting.
- Decoding of coded messages.
- Plotting of weather elements (wind, pressure, temperature etc.) on weather charts, for respective stations.
- Analysis of charts for different weather parameters: Drawing isolines of different weather parameters, viz., pressure, temperature etc.
- Once analysis is completed, zones with maximum value or minimum value of different parameters will be brought out clearly, known as synoptic features.
- Forecasters through their qualifications and professional trainings have the knowledge of weather associated with different synoptic features. For example: cold wave and fog occurs over a station, once a western disturbance passed away through it.
- Accordingly, a synoptic weather forecast bulletin is issued.

However, this method can give forecast for a broad region for a broad period. It involves lots of subjectivity, like knowledge, experience of the forecasters.
Location & time specific forecast is difficult.

Different components of a NWP model:

We know that weather $'WX'$at a place $(\hat{x})$ at a time $'t'$ is a function of the basic meteorological parameters, viz., zonal wind (u), meridional wind (v), vertical wind (w), pressure (p), temperature (T), specific/relative humidity (q) etc at that time over that place. Mathematically, $\mathrm{WX} = \mathrm{WX}(\mathrm{u}(\hat{\mathrm{x}},\mathrm{t}), \mathrm{v}(\hat{\mathrm{x}},\mathrm{t}), \mathrm{w}(\hat{\mathrm{x}},\mathrm{t}), \mathrm{P}(\hat{\mathrm{x}},\mathrm{t}), \mathrm{T}(\hat{\mathrm{x}},\mathrm{t}), \mathrm{q}(\hat{\mathrm{x}},\mathrm{t}))$. So, if the values of these basic meteorological parameters at a place $(\hat{\mathrm{x}})$ at a time $'\mathrm{t}'$ are known, then by establishing a suitable functional relation, as above, weather at the place $(\hat{\mathrm{x}})$ at time $'\mathrm{t}'$ can be determined. Thus the problem of forecasting weather has been translated to the problem of forecasting above meteorological parameters. Now question is whether it is possible to predict these parameters at a place for future. The answer to this question lies on the fact, whether these variables satisfy following type equation or not:

$$\frac{\partial \mathrm{f}(\hat{\mathrm{x}},\mathrm{t})}{\partial \mathrm{t}} = \mathrm{g}(\hat{\mathrm{x}},\mathrm{t});\ \text{given } \mathrm{f}(\hat{\mathrm{x}},\mathrm{t}_0) = \mathrm{f}_0(\hat{\mathrm{x}})$$

This class of equations, in mathematical literatures, are known as initial value problems (IVP). Fortunately all the above mentioned variables satisfy such initial value problem as given below:

$$\frac{\partial u}{\partial t} = -\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]u - \frac{1}{\rho}\frac{\partial p}{\partial x} - 2\Omega(w\cos\varphi - v\sin\varphi) + \frac{uv}{a}tan\varphi - \frac{uw}{a} + \frac{\mu}{\rho}\nabla^2 u$$

$$\frac{\partial v}{\partial t} = -\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]v - \frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega(u\sin\varphi) - \frac{u^2}{a}tan\varphi - \frac{vw}{a} + \frac{\mu}{\rho}\nabla^2 v$$

$$\frac{\partial w}{\partial t} = -\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]w - \frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega(u\cos\varphi) + \frac{u^2 + v^2}{a} + \frac{\mu}{\rho}\nabla^2 w$$

$$\frac{\partial T}{\partial t} = -\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]T + \frac{1}{c_v}\frac{dQ}{dt} - (\gamma - 1)T\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

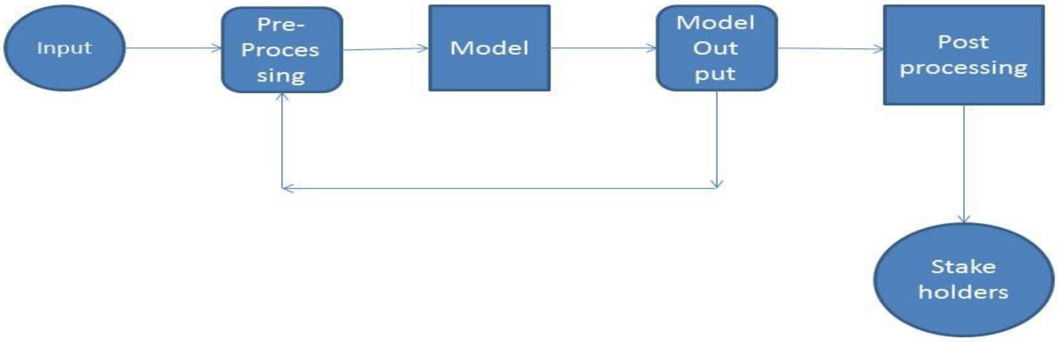
$$\frac{\partial \rho}{\partial t} = -\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]\rho - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

$$\frac{\partial q}{\partial t} = -\left[u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right]q$$

$$p = \rho RT$$

The above equations can be solved for future evolution of these variables, provided, values of these variables are known at initial time. In NWP, above equations are integrated forward with respect to time and the constants of integration are computed using the initial values of the variables.

Thus, without having the complete knowledge about these variables at initial time, we can’t have a particular solution for future values of these variables. In a typical NWP system, broadly there are three compartments, viz., pre-processing unit, model and the post processing unit, as shown schematically below:



In the pre-processing unit, observed data, available at unevenly spaced observing points, are subjected to different quality control checks, spatial and temporal consistency check and climatology check etc, followed by a sophisticated interpolation scheme to prepare values of the above variables at different grid points. It is most likely that grid point data, prepared in this way may contain errors, which is removed/minimised within a given tolerance limit and thus the initial values of the variables at grid points, i.e., initial conditions are prepared.

With above initial conditions, the equations are numerically integrated forward with time, in the Model. After integration, future values of the above variables at different grid points are generated, known as output.

These raw outputs may be of very little use for the stake holders, rather, stake holder may like to have specific weather information, like, rainfall, visibility, divergence, vorticity, precipitable water content etc, which are prepared by post processing of raw output from model. This post processing is being done in post processing unit.